# Numerical Modeling and Analysis of Transient Electromagnetic Wave Propagation and Scattering

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13. ABSTRACT *(Maximum 200 words)*

Our sponge layers are the only ones for which a proof of the reflectionless property has been given in cylindrical and spherical coordinates. More significantly, we have obtained initial results (presented at the 14th ACES conference) that indicates our sponge layer in spherical coordinates produces the same error as the exact ABC but at a fraction of the computational cost. We are continuing with analysis and numerical comparisons with exact ABC's in ABC's instead of the simpler Dirichlet boundary condition to terminate the sponge layers in the time-domain is desirable only for shallow layers and for layers of any depth but with a constant loss profile. Our efforts to determine the numerical reflection of the discrete sponge layer in cylindrical and spherical coordinates necessitated the derivation and study of the numerical dispersion relation of the discrete wave equation in these two curvilinear coordinate systems. With this work we have for the first time derived guidelines to limit the phase error when using finite difference schemes in curvilinear coordinates and have opened the avenue towards determining the numerical reflection of a given layer in such coordinate systems and optimizing it.

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# 1 Technical Summary of Research Accomplished

Objectives

a) Our approach to the derivation of reflectionless sponge layers can also be viewed as a mapping of dielectrics that does not require field splitting to produce time-domain equations. We will study such mappings in the three- dimensional cylindrical and spherical coordinates in the frequency- and time-domains.

b) The sponge layer/free-space interface is reflectionless for analytic waves, however, discrete waves may partially reflect from it. For this reason we wish to rigorously study this reflection and to determine appropriate ABCs for use in place of the PEC boundary condition currently used in the truncation of perfectly matched and reflectionless sponge layers.

Status

a) We have derived the unsplit reflectionless sponge layers for the frequency- domain (elliptic) and time-domain (hyperbolic) Maxwell's equations. We have proven the well-posedness of our formulation in all coordinate systems. Using Mie series we have rigorously proved that our layers are reflectionless in cylindrical and spherical coordinates. We have implemented our approach in pseudo-spectral codes and have compared it to alternative formulations proposed by other researchers (joint work with B. Yang, Brown University). Also, we have implemented the three- dimensional spherical case in a finite difference code and have compared our approach to that of Grote and Keller which is a time-local implementation of the exact ABC in spherical coordinates (joint work with N. kantartzis).

b) We completed a theoretical and numerical investigation of the behavior of the ABC-backed reflectionless sponge layer in rectangular coordinates. Also, work was initiated to study the numerical reflection of such layers in cylindrical and spherical coordinates.

Findings

a) Our sponge layers are the only ones for which a proof of the reflectionless property has been given in cylindrical and spherical coordinates. More significantly, we have obtained initial results (presented at the 14th ACES conference) that indicate our sponge layer in

spherical coordinates produces the same error as the exact ABC but at a fraction of the computational cost. We are continuing with analysis and numerical comparisons with exact ABC's in cylindrical and spherical coordinates.

b) We have determined that using local ABC's instead of the simpler Dirichlet boundary condition to terminate the sponge layers in the time-domain is desirable only for shallow layers and for layers of any depth but with a constant loss profile. Our efforts to determine the numerical reflection of the discrete sponge layer in cylindrical and spherical coordinates necessitated the derivation and study of the numerical dispersion relation of the discrete wave equation in these two curvilinear coordinate systems. With this work we have for the first time derived guidelines to limit the phase error when using finite difference schemes in curvilinear coordinates and have opened the avenue towards determining the numerical reflection of a given layer in such coordinate systems and optimizing it.

# 2 Conference Papers

"The Unsplit PML for Maxwell's Equations in Cylindrical and Spherical Coordinates," in *Proceedings of the 14th Annual Review of Progress in Applied Computational Electromagnetics*, vol. II, pp. 615-622, 1998. Paper was presented in a special session organized by the PI titled "ABC's for CEM: Theoretical and Implementation Issues".

"A Comparison of the Grote-Keller and Unsplit PML Absorbing Boundary Conditions for Maxwell's Equations in Spherical Coordinates," in *Proceedings of the 14th Annual Review of Progress in Applied Computational Electromagnetics*, vol. II, pp. 623-680, 1998. Paper was presented in a special session organized by the PI titled "ABC's for CEM: Theoretical and Implementation Issues".

"Well-posed Perfectly matched Layers for the Numerical Solution of Maxwell's Equations in Rectangular, Cylindrical, and Spherical Coordinates," in *Mathematical and Numerical Aspects of Wave propagation*, J. A. Desanto, Ed., pp. 567-569, 1998.

# 3 Presentations

"Perfectly Matched Sponge Layers as ABCs for the Numerical Solution of Maxwell's Equations in Rectangular, Cylindrical, and Spherical Coordinates," presented at:

Department of Mathematical Sciences, University of Delaware, October 1997.
USAF Electromagnetics Workshop, Brooks AFB, January 1998.
Center for Computational Electromagnetics, University of Illinois at Urbana-Champaign, February 1998.

"Reflectionless Sponge Layers as ABCs for the Numerical Solution of Maxwell's Equations in Rectangular, Cylindrical, and Spherical Coordinates," presented at:
Department of Applied Mathematics, Cal-Tech, March 1998.
14th Annual Review of Progress in Applied Computational Electromagnetics, March 1998.

The above was also presented as an invited talk in the Ryaben'kii Workshop, International Conference on Spectral and High Order Methods, Israel, June 1998.

"A Comparison of the Grote-Keller and Unsplit PML Absorbing Boundary Conditions for Maxwell's Equations in Spherical Coordinates," presented at 14th Annual Review of Progress in Applied Computational Electromagnetics, March 1998.

# 4 Consultative And Advisory Functions To Other Laboratories And Agencies

Throughout the period covered by this report I continued my interaction with Dr. T.M. Roberts (Rome Laboratory/ERAA, Hanscom AFB) regarding finite difference solvers for the time-domain Maxwell equations.

Also, Dr. H. Steyskal (Rome Laboratory/ERAA, Hanscom AFB) considered allowing his graduate student an extended visit to my institution in order to collaborate on numerical methods for the solution of practical electromagnetic problems of common interest. However, the student decided to go elsewhere.

The PI has completed his contribution (Chapter on FD-TD) to the Springer-Verlag volume that will contain the proceedings of the 1997 NATO Advanced Study Institute (Greece) on Applied Computational Electromagnetics: State of the Art and Future Trends.

The PI has been invited to edit a special issue on "ABC's for CEM" for the International Journal of Numerical Modelling: Networks, Circuits and Fields." The announcement appeared in NA-NET and in IJNM, Vol 11, issue 4 (July/August 1998).

# References

[1] P. G. Petropoulos, L. Zhao and A. C. Cangellaris, "A Reflectionless Sponge Layer Absorbing Boundary Condition for the Solution of Maxwell's Equations with High-Order Staggered Finite Difference Schemes," *J. Computational Physics*, vol. 139, pp. 184-208, 1998

[2] P. G. Petropoulos, "On the Termination of the Perfectly Matched Layer with Local Absorbing Boundary Conditions," *J. Computational Physics*, vol. 143, pp. 665-673, 1998.

[3] B. Yang and P. G. Petropoulos, "Plane-Wave Analysis and Comparison of the Split-Field, Biaxial and Uniaxial PML ABC Methods for Pseudospectral Electromagnetic Wave Simulations in Curvilinear Coordinates," *J. Computational Physics*, vol. 146, pp. 747-774, 1998.

[4] P. G. Petropoulos, "Numerical Dispersion and Absorbing Boundary Conditions," *International Journal of Numerical Modelling: Networks, Circuits and Fields*, to appear in vol. 13, no. 3.